A Mathematical Model of Robust Military Village Searches for Decision Making Purposes

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Abstract - *In the modern, fast-paced, high technology military, making decisions on how to best utilize resources to accomplish a mission with a set of specified constraints is difficult. A Cordon and Search of a village (i.e., village search) is an example of such a mission. Leaders must plan the mission, assigning assets (e.g., soldiers, robots, unmanned aerial vehicles, military working dogs) to accomplish the given task in accordance with orders from higher headquarters. Computer tools can assist these leaders in making decisions, and do so in a manner that will ensure the chosen solution is within mission constraints and is robust against uncertainty in environmental parameters. Currently, no such tools exist at the tactical or operational level to assist decision makers in their planning process and, as a result, individual experience is the only tool available. This paper proposes a methodology and a mathematical model for village searches that applies robustness concepts resulting in a decision-making tool for military leaders to use in mission planning.[1]*

Keywords: robustness, resource allocation, village search, decision making, stochastic processes

1 Introduction

On the modern battlefield, village searches are a critical mission for military ground forces. In the current Global War on Terrorism environment, these searches are conducted daily to clear villages, capture insurgents, confiscate contraband, etc. When planning a village search, military staff officers must analyze the problem, allocate resources to the mission, estimate the amount of time required to complete the mission, plan for contingencies, and publish a mission plan. Frequently, a unit tasked with a village search mission is given a time constraint for the completion of the mission. This constraint affects the development of the plan and the manner in which it is conducted. To add to the complexity of the planning, participating elements can include: soldiers, Military Working Dogs (MWD), Explosive Ordinance Detachments (EOD), military aircraft, Unmanned Aerial Vehicles (UAVs), and electronic surveillance. Given the diversity and the unpredictability of the battlefield, it is an arduous task to develop a robust plan for a village search.

Despite the high frequency of this mission type, no automated tools currently exist to assist military leaders in planning the execution of the searches. To develop their plans, officers must rely on the experience they have gathered during their years of service and the limited data tables provided in military Field Manuals (e.g., [8], [9]) for factors such as ground movement rates. As a result, the quality of a plan, its ability to account for uncertainty, and the resulting confidence in its success varies dramatically.

An automated tool for village search planning would greatly assist military leaders in their decision making process in this environment where lives are at risk. To be effective, the tool requires the following capabilities: model the search area using existing input imagery files; use probabilistic models to calculate search times; determine the most robust solution from multiple possible resource allocations; and execute within the time constraints of the planning process. The long term goal for this tool is to allocate resources (e.g., humans, MWDs, EOD, UAVs) to tasks (e.g., building searches), and calculate the robustness for the resource allocation, which is the probability of mission completion within the time constraint.

The contributions of this paper are a methodology and a mathematical model for village searches. This is the foundation for a resource allocator, based on a stochastic model that is robust against system uncertainty. The methodology describes how village searches can be modeled with uncertainty incorporated. The mathematical model allows for the objective evaluation of resource allocations and thus the selection of an allocation that results in an acceptable plan based upon the computation time required and the amount of computing resources used.

This paper assumes that the probability distribution functions (pdfs) for the quantities that are uncertain can be developed. The definition of these pdfs is a separate research problem and is not addressed here. Additionally, the model assumes that only one search team is used per target building.

[1] This research was supported by the NSF under grant No. CNS-0615170 and by the Colorado State University George T. Abell Endowment.

Future research on this problem will remove these assumptions.

With these goals in mind, Section 2 of this paper provides a discussion of the robustness metric developed in [2, 12]. In Section 3, a model for a basic village search problem is presented. Section 4 reviews research related to the village search problem. An outline of future research work that builds on the robust village search model is in Section 5. Finally, in Section 6 we present the conclusions.

2 Robust resource management

We have defined robustness and a methodology to calculate the robustness of a resource allocation in [1] and studied it within a variety of systems (e.g., [2, 12, 13]). We propose to adapt the concept of robustness to the problem of village search planning.

The robustness metric for a given resource allocation can be developed using the FePIA (Features, Perturbation parameters, Impact, Analysis) method [1], where the following are identified: (1) the performance features that determine if the system is robust, (2) the perturbation parameters that characterize the uncertainty, (3) the impact of the perturbation parameters on the performance features, and (4) the analysis to quantify the robustness. The FePIA method provides a formal mathematical framework for modeling the dynamic village search environment.

The performance features are those measurable system attributes that can be compared against the robustness criteria. For a village search this can be the time required for a search team to finish searching its assigned buildings. That is, if there are m search teams, there are m performance features where it is the time search team i finishes searching its assigned buildings. For the system to be robust, all search teams must complete before the mission time constraint.

Perturbation parameters (uncertainties) that may affect the actual mission completion time include: weather, estimation error in search area dimensions, variability in search resource movement rates, frequency and number of casualties, equipment losses, and number of enemy combatants encountered. Each of these elements can impact the actual mission completion time in a positive or negative manner, but the key point is that their actual values at the time of the mission are uncertain when planning is done. The system must account for these perturbations and recommend a resource allocation that is robust with respect to these uncertainties.

Once the perturbation parameters are enumerated, it is necessary to describe mathematically how they impact the performance features. For example, a stochastic model may be used to describe the effects of enemy combatants on the search of a given building. The collective effects of the uncertain perturbation parameters on the performance features must then be evaluated to find the most robust allocation of assets.

For the final step, stochastic (probabilistic) information about the values of these parameters whose actual values are uncertain is used to quantify the degree of robustness. The resulting stochastic robustness metric (*SRM*) is the probability that a user-specified level of system performance can be met. In this domain, the performance metric is the completion time for the search of the whole village by the slowest search team.

Using these FePIA steps, the stochastic robustness metric for a village search can be determined. Once this is done, heuristics for planning robust resource allocations can be designed. Furthermore, during the mission, heuristics can be utilized to dynamically reallocate resources as the situation on the ground changes from the initial assumptions. This paper focuses on the development of the mathematical model needed for calculating the robustness of a plan; future work will use this model to design resource allocation heuristics.

3 Village search model

A quantitative mathematical model for a village search is presented in this section. To illustrate the problem, Figure 1 provides an example allocation for a village search scenario. As shown in the figure, a village is comprised of a set of target buildings, $T = \{T_1, T_2, \ldots\}$ and a set of movement paths $M = \{M_1, M_2, \ldots\}$ with associated distances between buildings. These target buildings are assigned for search using search resources, $R = \{R_1, R_2, \ldots\}$, where R_i can represent a human search team, a military working dog team, an Explosive Ordinance Detachment, a robot, etc. Military planners must allocate the resources (search resources) to the tasks (building searches) in a manner that will meet the given performance requirement (village search mission deadline time). A model of this scenario must account for factors such as the search rate of the search resources, the movement time between structures, the ordering of the structure searches, and the perturbations discussed in the previous section.

To apply the robustness procedure to the village search scenario, one must answer the three robustness questions in [2]. Namely: (1) What behavior of the system makes it robust? (2) What uncertainties is the system robust against? and (3) Quantitatively, exactly how robust is the system? In this case, the system is defined as the village search mission with its assigned search resources, target buildings, and the environment. The required behavior for the system to be considered robust may be one of or a combination of criteria, such as, a specified time constraint is met, a specified percentage of casualties or less occurs, or no high value equipment is destroyed. For the development of this model, the robustness criterion is the Mission Deadline Time (*MDT*) or time by which the mission must be completed.

A system of this type will need to be robust against a variety of dynamic uncertainties that occur in the field including the number of enemy combatants encountered, weather, encounters with explosive hazards, treatment and evacuation of casualties, changes to the availability of resources, and unanticipated animal (MWD) behavior. This paper initially considers the following perturbation parameters: temperature (heat), $\underline{H}$; precipitation, $\underline{P}$, variability in the base search rate, $\underline{B_{Ri}}$, for search team i; and estimation error in the dimensions of the search area, $\underline{A_E}$. A list of notation used in this model is shown in Table 1. Stochastic models can be developed for each of these perturbation parameters but further study is needed to develop these pdfs (this is a separate problem that may be approximated experimentally).

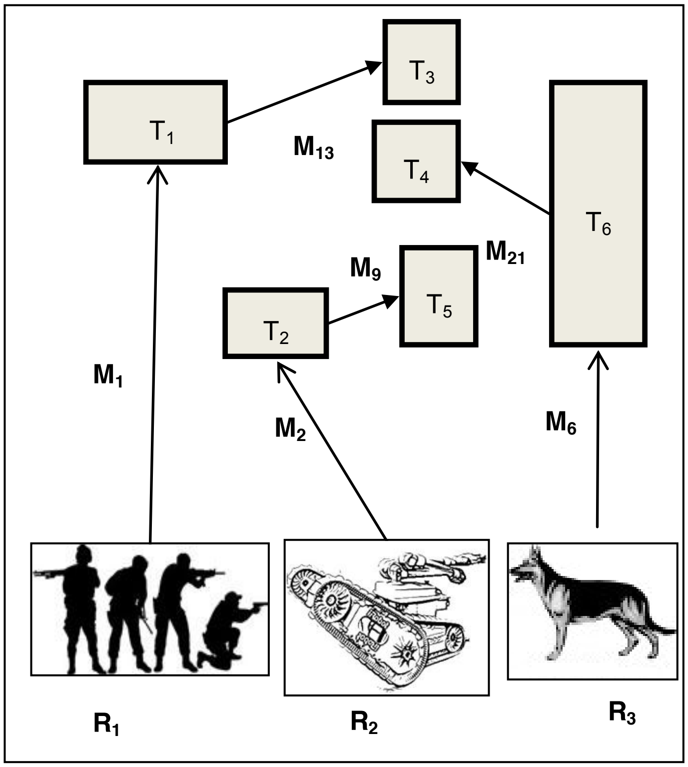

Figure 1. An example resource allocation for a village search with three search resources (human team, robot team, and a military working dog team) allocated to six tasks (building searches) with six movement paths.

In the village search scenario, these perturbations are modeled as follows. Future temperature values have been modeled using normal distributions [5, 6], so H will be modeled as a pdf that maps weather variables to the probability of a future temperature value. This future temperature value can then be input as a variable into the completion time function. Future precipitation amounts, which also can decrease the search rate due to soil conditions and visibility limitations, have been modeled using Markov chains (probability of rain) and Gamma distributions (quantity of rainfall) [6]. Similar to H, P is a future precipitation pdf that maps weather variables to the probability of a future precipitation value. Errors in the estimation of building presence and size have also been formulated [10, 15]. Finally, a base search rate distribution can be developed by gathering data from observation of training exercises or combat operations. This base search rate is used as an input variable to the overall completion time function.

To make determinations on resource allocations with regard to robustness, a quantitative method for calculating robustness is required. Applying the general stochastic model of robustness developed in [12], this is defined as the probability that a user-specified level of system performance can be met. Let the maximum search resource completion time for the set of search teams be $\underline{RCT_{max}}$. Then the robustness requirement is $RCT_{max} \leq MDT$.

Table 1: Village search tool nomenclature.

Name	Description
	Robustness Variables
SRM	stochastic robustness metric, probability that the village search completion time is less than MFT
RCT_i	resource completion time for team i
RCT_{max}	maximum resource completion time for all teams
	Variables
A_j	area of building j
A_{Ej}	estimation error of area of building j
B_{Ri}	base search rate for search resource i
C_{ijk}	completion time for team i on building j and movement leg k
G_{Ri}	ground movement rate for search resource i
H	future temperature pdf
MDT	mission deadline time
M_k	movement path k leading to building j
P	future precipitation pdf
R_i	search resource i
T_i	target building i

A village consists of a set of target buildings, with a corresponding set of areas, $\underline{A} = \{A_1, A_2, \ldots\}$. These areas include multiple floors of the target building. Let $\underline{A_{Ej}}$ be defined as the dimensional error of building A_j. The resource's ground movement rate, $\underline{G_{Ri}}$, is the rate that the resource can move tactically along a movement path M_k. Therefore, completion time, $\underline{C_{ijk}}$, for resource i searching a given target building T_j and traversing movement path k is simply the area of the building divided by the search rate plus the distance of the movement path to the building divided by the ground movement rate. The completion time is a function of the perturbation parameters: A_{Ej}, A_j, B_{Ri}, M_k, H, P, and G_{Ri}. Finally, the subset of target buildings assigned to a search resource and the order of search is constrained by tactical considerations (e.g., the maximum effective range of the weapons systems in the supporting elements) and thus some orderings are invalid. The perturbation parameters, elements such as A_{Ej}, B_{Ri}, H, P, and G_{Ri}, are random variables. Given these random variables, the completion time for team i on target building j and its corresponding movement path k has a distribution function defined as:

$$C_{ijk} = f_{C_{ijk}}\left(A_{Ej}, A_j, B_{Ri}, M_k, H, P, G_{Ri}\right). \quad (1)$$

Equation 1 results in a random variable with a distribution consisting of building search completion times. It is assumed that the pdf for this function will be created at run time using input values for the perturbation parameters (e.g., probability and amount of rainfall, predicted temperature).

Summing the building completion times for a resource results in the resource completion time, RCT_i, for a search resource, R_i, where k is the movement path associated with T_j and n is the number of target buildings in its search set.

$$RCT_i = \sum_{j=1}^{n} C_{ijk}. \quad (2)$$

Define the probability that search resource R_i finishes searching its target set in less time than the MDT, where '$*$' represents convolution, by the equation:

$$P(RCT_i < MDT) = \int_{-\infty}^{MDT} f_{C_{i1k}} * f_{C_{i2k}} * \cdots * f_{C_{ink}}. \quad (3)$$

Initially it is assumed that the search resources have adequate supporting elements to operate independently. Additionally, the perturbation parameters considered are independent with respect to the search resources and therefore the resource completion times are independent. This allows the stochastic robustness metric SRM (with m the number of search resources) to be defined as:

$$SRM = \prod_{i=1}^{m} \mathrm{P}(RCT_i < MDT). \quad (4)$$

Thus, for a given resource allocation of search resources to target buildings, the SRM provides the quantitative value for the robustness of the allocation. Therefore, the possible set of allocations can be searched to determine the allocation that is most robust via the comparison of SRM values.

Building on the general discussion in [14], the robustness metric can be utilized in two manners for the village search tool. In the first scenario, a military unit is tasked to conduct a village search within a given time constraint. Here the tool is used to calculate the resource allocation that has the highest probability of meeting the mission deadline time. For the second scenario, a military unit is tasked to search a village and requires an accurate estimate of the maximum resource completion time to allow for the planning of supporting assets. In this case, the robustness metric is used to calculate the maximum resource completion time for the mission with a given probability (e.g., 95%). To accomplish the operational goals of the village search tool, future research is required in many areas as outlined in Section 5.

4 Related work

The related literature on the topic of village searches covers two areas: basic data look-up tables [8, 9] and military simulations. These works contain elements related to the tool proposed here, but do not have a similar scope or functionality. The existing models are either more fine grained, i.e., modeling individual soldier movement, or are large grained and focused on the Corps and higher level movements and combat resolution. The village search tool proposed here focuses at a medium grained level for company, battalion, and brigade size operations.

As mentioned previously, military Field Manuals, such as Army FM 3-31.1 and FM 34-8-2, provide simple deterministic tools for planners to use when developing mission directives. One example is a data table utilized for determining the ground movement rate of soldiers in combat. In this table, planners must choose from four environmental conditions to produce the output ground movement rate. Our model uses a stochastic method for determining ground movement rates and accounts for perturbations that affect the search rate, thus creating a more realistic and flexible model.

Much work has been done in the field of military combat simulations. Most of the simulations are based on deterministic models [3, 4] though work has been done on stochastic models [7, 11]. The purpose of these simulations generally fall into two categories: training aids for troops or strategic-level simulations for theater plans. Within some of these simulations, urban movement and combat at the soldier level is modeled, but it is not for the purpose of resource allocation and decision making. The village search tool differs from these simulations by providing a resource allocation or a mission deadline time planning value that assists the military leader in making decisions. Finally, the urban or village models rely on deterministic look-up tables for their input data or stochastic models that account for randomness in only movement direction and combat strategy. The village search model utilizes stochastic methods to account for perturbation parameters and thus provides a robust resource result.

5 Future work

The proposed mathematical model for the village search mission is a decision making tool. To maximize the utility of the model, it is necessary to account for as many of the possible perturbations as feasible. This requires research to define the pdfs for the perturbation parameters. Data for the base search rates and the effects of temperature and precipitation on the rates can be collected from the military's training centers or from collected combat data if it is not classified. The large number of mission repetitions conducted in those environments provides enough sample points to construct a reliable distribution.

Additionally, the resources available for assignment need to include MWD teams, specialist search teams, manned and unmanned aircraft, EOD teams, and, as systems develop, robots. Resource movement rates, resource movement techniques, and the interaction among resources all need to be modeled.

Further research is needed in the calculation of the estimation error in building size from imagery. Current work mostly focuses on the error in the detection of buildings and not the error in all three dimensions. A complete model would provide an estimate of possible subterranean area and be able to subtract portions of the structure that were inaccessible due to obstructions or partial building collapse.

All of the above proposed future work will build on the framework presented here for the purpose of designing allocation heuristics. We will combine knowledge from the literature on resource management techniques for heterogeneous computing systems (e.g., [2]) and on military operations to produce such heuristics. These heuristics will be employed to create robust plans for allocating search teams to buildings to perform a village search. The heuristics will be executed prior to the deployment of the teams, and thus come under the category of "static" or "off-line" resource allocation heuristics (e.g., [12]).

The ability to dynamically update the resource allocation as a mission progresses is another area for research. As the village search mission progresses and conditions change, the ability to re-evaluate the resource allocation or the estimated *RCT* would be invaluable to the ground force commander. Part of the work required includes determining the appropriate triggers for executing a dynamic reallocation. In this scenario, the execution time of the tool would be a key factor.

Finally, the completed model requires field testing. Again, this could be done at the training centers or with units in combat. The training centers provide an excellent means of testing the tool in an unobtrusive manner while also minimizing risk should the tool be inaccurate.

6 Conclusions

Village searches are an integral component of the military's ground combat operations and will continue to remain so for the foreseeable future. The difficult conditions and the constraints imposed on the planning for these types of missions create a large variability in the quality of the mission plans. A decision making tool that can assist military leaders in the development of a village search mission plan can reduce the uncertainty and risk inherent in combat operations. Ultimately, this would increase the effectiveness of our ground combat troops and reduce casualties of all types.

A mathematical model and a methodology for evaluating the robustness of resource allocations in a village search scenario were presented in this paper. The model accounts for the uncertainty in the perturbation parameters and produces a quantitative measure of the allocation's stochastic robustness with regard to the performance feature. Unlike the majority of planning tools and military simulations currently in existence, the tool uses stochastic information to model what is naturally an extremely variable environment – combat.

Acknowledgments

The authors would like to thank Dr. T. Lucas of the Naval Postgraduate School for his assistance with background references and Dr. A. Rosenberg, L. Briceño, and A. Al-Qa'wasmeh for their comments.

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